Vulnerable American options
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Abstract
Purpose – The purpose of this paper is to extend the models of Johnson and Stulz, Klein and Klein
and Inglis to analyse the properties of vulnerable American options.
Design/methodology/approach – The presented model allows default prior to the maturity of the
option based on a barrier which is linked to the payoff on the option. Various measures of risk
denoted by the standard Greek letters are studied, as well as additional measures that arise because
of the vulnerability.
Findings – The paper finds that the delta of a vulnerable American put does not always increase with the
price of the underlying asset, and may be significantly smaller than that of a non-vulnerable put. Because
of deadweight costs associated with bankruptcy, delta and gamma are undefined for some values of the
underlying asset. Rho may be considerably higher while vega may be smaller than for non-vulnerable
options. Also, the probability of early exercise for vulnerable American options is higher and the price of
the underlying asset at which this is optimal depends on the degree of credit risk of the option writer.
Originality/value – This paper makes a contribution to understanding the effect of credit risk on option
valuation.
Keywords Derivative markets, Diffusion, United States of America, Pricing, Stock options
Paper type Research paper

1. Introduction
It is well known that the value of an over-the-counter option is reduced when the payoff
is vulnerable to the credit risk of its writer. This is because the full payoff may not be
made if the writer is experiencing financial distress.

The impact of credit risk on the value of debt has been well studied since the early
days of modern option pricing theory (e.g. Black and Scholes, 1973; Merton, 1974; Black
and Cox, 1976; Shimko, et al. 1993; Longstaff and Schwartz, 1995). As yet there has
been much less research completed on the effect of credit risk on the value of options.

Johnson and Stulz (1987) were the first to develop a model which values European
options subject to default risk at the maturity of the option. Under the assumption that
the option is the only liability of the option writer and the option holder receives all the
assets of the option writer if default occurs, they demonstrate that the effect of credit
risk can be significant. They also analyse various additional considerations for
American options, such as the effect on the optimality of early exercise.

Hull and White (1995) assume that when the option writer defaults there are other
equal ranking claims in addition to the option. Default can happen at any time before
the option expires. The default boundary is fixed and default is assumed to occur
whenever the value of the assets of the option writer falls below that fixed level. In the
event of default, the option holder receives a proportion of the nominal amount of the
claim. Their numerical examples show that the impact of credit risk on American
options is much less than for European options.

Klein (1996) extends the model of Johnson and Stulz (1987) by allowing the option
writer to have liabilities other than the option and by allowing the recovery rate to
depend on the value of the assets of the option writer. These assumptions are
more appropriate when applying the model to many business situations. He derives a
closed-form solution for European options and shows that the model is easy to calibrate.