



# Vulnerable American options

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## Abstract

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**Purpose** – The purpose of this paper is to extend the models of Johnson and Stulz, Klein and Klein and Inglis to analyse the properties of vulnerable American options.

**Design/methodology/approach** – The presented model allows default prior to the maturity of the option based on a barrier which is linked to the payoff on the option. Various measures of risk denoted by the standard Greek letters are studied, as well as additional measures that arise because of the vulnerability.

**Findings** – The paper finds that the delta of a vulnerable American put does not always increase with the price of the underlying asset, and may be significantly smaller than that of a non-vulnerable put. Because of deadweight costs associated with bankruptcy, delta and gamma are undefined for some values of the underlying asset. Rho may be considerably higher while vega may be smaller than for non-vulnerable options. Also, the probability of early exercise for vulnerable American options is higher and the price of the underlying asset at which this is optimal depends on the degree of credit risk of the option writer.

**Originality/value** – This paper makes a contribution to understanding the effect of credit risk on option valuation.

**Keywords** Derivative markets, Diffusion, United States of America, Pricing, Stock options

**Paper type** Research paper

## 1. Introduction

It is well known that the value of an over-the-counter option is reduced when the payoff is vulnerable to the credit risk of its writer. This is because the full payoff may not be made if the writer is experiencing financial distress.

The impact of credit risk on the value of debt has been well studied since the early days of modern option pricing theory (e.g. Black and Scholes, 1973; Merton, 1974; Black and Cox, 1976; Shimko, *et al.* 1993; Longstaff and Schwartz, 1995). As yet there has been much less research completed on the effect of credit risk on the value of options.

Johnson and Stulz (1987) were the first to develop a model which values European options subject to default risk at the maturity of the option. Under the assumption that the option is the only liability of the option writer and the option holder receives all the assets of the option writer if default occurs, they demonstrate that the effect of credit risk can be significant. They also analyse various additional considerations for American options, such as the effect on the optimality of early exercise.

Hull and White (1995) assume that when the option writer defaults there are other equal ranking claims in addition to the option. Default can happen at any time before the option expires. The default boundary is fixed and default is assumed to occur whenever the value of the assets of the option writer falls below that fixed level. In the event of default, the option holder receives a proportion of the nominal amount of the claim. Their numerical examples show that the impact of credit risk on American options is much less than for European options.

Klein (1996) extends the model of Johnson and Stulz (1987) by allowing the option writer to have liabilities other than the option and by allowing the recovery rate to depend on the value of the assets of the option writer. These assumptions are more appropriate when applying the model to many business situations. He derives a closed-form solution for European options and shows that the model is easy to calibrate.

