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From Commercial Arithmetic to Life Annuities: The Early History of Financial Economics, 1478-1776.*

Geoffrey Poitras

Faculty of Business Administration
Simon Fraser University
Burnaby, B.C.
CANADA V5A 1S6

and

Department of Economics and Statistics
National University of Singapore
10 Kent Ridge Crescent
SINGAPORE 0511

ABSTRACT

This paper considers the early history of financial economics, focusing on the origins of security pricing theories. The period under examination, 1478-1725, starts with the first printed commercial arithmetic and ends with de Moivre's contributions to the pricing of life annuities. Much of the relevant material is unavailable in conventional sources on the history of economics and is not systematically addressed by histories of mathematics, statistics and science. Unlike other areas of economic analysis, significant theoretical results had been achieved for pricing certain securities by the middle of the 18th century. Subjects examined include the development of commercial arithmetic including compound interest problems, the substantial results achieved for the pricing of life contingent claims, and the relatively less developed theories for pricing joint-stocks and derivative securities.

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Compared to other areas of modern economic analysis, relatively little has been written which is directly concerned with the early history of financial economics. One reason for this situation is the limited attention given to **security pricing** problems by economists writing prior to the modern era. Many of the essential early contributions related to pricing the available securities were made either by mathematicians or anonymous merchants involved in related commercial activities. By the early eighteenth century, substantial theoretical results had been developed for pricing the sometimes complicated financial securities which were available. The contrast between the advanced development that certain aspects of financial economics had reached by this time with the rudimentary state of the rest of economic theory is striking. The primary objective of this paper is to provide an examination of the numerous individuals and problems involved in the early history of security pricing, which may be unfamiliar to historians of economic thought. Much of the requisite information is not systematically addressed by histories of mathematics, statistics and science which, understandably, have been more concerned with discussing the problems which are of interest to those fields of study.

Examination of the early history reveals the fundamental influence of canon law on security pricing practices. The evolution of scholastic doctrine on usury and gambling, the licit partnership, and the triple contract legitimized the progressive weakening of religious controls on the accumulation of capital and contributed directly to changing practices for pricing financial instruments used in both business and government activities. Important contributions to security pricing often originated from individuals operating outside the scholastic realm: the early reckoning masters and commercial algorists such as Chuquet, the Dutch prime minister de Witt, up to the algorist Abraham de Moivre. Over time, numerous mathematical and statistical techniques were adapted to solve practical financial problems. The early history reveals considerable interaction between contributions to pure mathematics and applied contributions to security pricing, such as Huygens' theoretical contribution on mathematical expectation leading to de Witt's pricing formula for a life annuity and Chuquet's concern with solutions of compound interest problems leading to development in algebraic methods.

Modern financial economics is a large subject area. Many financial theories of contemporary interest, such as decision making under uncertainty, portfolio management and the term structure of interest rates, were not developed or recognized until well beyond the period examined in this study. In addition, much of modern financial economics would have been too scholarly and, as a result, uninteresting to the early security market participants who were more interested in practical and readily implemented rules for determining prices of extant securities. To provide some relevant background, in the following Section 1 contains an overview of the commercial history for the period under consideration, focusing on the types of securities traded. The impact of canon law on commercial practices is also discussed. Section 2 examines the early commercial arithmetics. The related problems of determining simple interest in partnerships and compound interest are directly considered. Section 3 is concerned with the evolution of interest calculations into the modern equivalents of discounting and present value. The social changes which facilitated this evolution are considered. Section 4 deals with the valuation of life annuities. An incongruity between general commercial practice and available theoretical results is identified. Section 5 deals with the pricing of joint-stocks and derivative securities. The rudimentary state of pricing for these securities is discussed. Finally, Section 6 summarizes the main results contained in the paper.

1. Background on the History of Commerce[1]

While it is possible to trace the origins of early financial economics back to antiquity, the starting point for this study is the publication of The Treviso Arithmetic (1478), the earliest known printed mathematics book in the West. The Treviso is a commercial arithmetic, a type of early business school textbook (Swetz 1987). In addition to developing the requisite arithmetical calculations, much of the Treviso is concerned with solving problems arising in medieval commerce: currency exchange, calculating payment for exchanging goods, and determining shares of profit derived from partnerships. The types of problems considered in the early commercial arithmetics, as well as the associated solutions, are an important source of information about business practices of the times. For example, consider the emphasis on problems of dividing the shares from partnerships. Throughout the earlier history, prohibitions against usury and gambling had a significant impact on the recognition and valuation of interest payments (e.g., Noonan 1957, Daston 1988, chp.1). As well as being a primary source of funds for business enterprises, because income received from partnerships was considered licit under canon law, the partnership was also used as a method of disguising interest payments in order to avoid the usury prohibition.

Commercial arithmetic was concerned with the mathematical methods and problems encountered by merchants of the time. The commercial arithmetics included 'security' valuation problems involving interest calculations, where the precise form of the interest calculations depended fundamentally on the avoidance of usury, a subject which was governed by canon law. More precisely, under canon law *interisse* (from the Latin verb "to be lost") was acceptable while *usura* (from the Latin noun "use") was not. Compensation could be charged for a loan (*mutuum*) only if it was a reimbursement for a loss or expense, no net gains were permitted (e.g., Dempsey 1948). Various interpretations of canon law permitted interest to be paid on state loans, partnerships, and the census. Interest was also disguised in monetary exchange transactions combined with credit which took the form of bills of exchange (e.g., Einzig 1970). The interest derived from partnerships led to the development of interest on bank deposits and, starting around 1485 (Noonan 1957), to the "triple contract", an 'insured' partnership with a fixed rate of return. The interaction between the growth of commercial activity and social acceptance of interest payments is an essential element in the evolution of security pricing theories.

Commercial arithmetic was often taught by reckoning masters operating outside the universities. Reckoning schools, where reckoning masters accepted students for private tuition or conducted formal group classes, appeared in Italy prior to the 14th century and spread to northern Europe along the trade routes. Students came, typically, from merchant families, usually following a grammar school education. The reckoning school functioned as an essential component in the training of merchant apprentices. The reckoning master also acted as a consultant on various types of often complicated problems involving commercial calculations. While Italy was the centre for European commercial education, by the time of the printing of the Treviso there were numerous important commercial centres outside of Italy where reckoning masters, or algorists, were in increasing demand. Lyon was one such centre, where a flourishing spice market contributed to the success of the Lyon fairs following the royal protection provided to the fairs by Louis XI in 1464. It was in Lyon where Nicholas Chuquet, a master algorist, worked and by 1484 completed a series of manuscripts referred to as the Triparty.

The problems contained in the commercial arithmetics provide important insights into the business practices of the time. The growth of markets, the Reformation, and a host of other factors contributed to the gradual evolution of financial securities, such as the census, and, in turn, to the evolution of security pricing techniques. The census was a form of investment dating at least to feudal times.[2] These contracts were an obligation to make annual payments, secured by either land or the taxing power of the state. Initially designed as a type of barter arrangement, present goods for future goods, the census gradually took the form of a modern annuity where cash was received by the seller of the annuity in exchange for an agreement to make a stream of annual payments over time. By the time of the Treviso, the nobility, the church, the state and the landed gentry were all involved as sellers of census. There were many different variations of census: a life census in which payments were made over the life of a buyer, or their designee; a perpetual census, which had no fixed maturity date; and, a temporary or term census which ran for a fixed number of years, similar to a mortgage. A census could have conditions which permitted it to be redeemable at the option of either the buyer or seller.

Over time, certain types of census became securitized and negotiable. As early as the 13th century, Venice issued a census, the prestiti, offering 5% annual payment with no stated maturity date which was negotiable and was partially redeemed when Venetian finances permitted. By the 14th century, the Council of Venice permitted designated foreigners to purchase prestiti and the security was conveyed to other major European centres, being considered a "much sought after" secure investment (Homer and Sylla 1991). The demand for prestiti reflected the limited number of secure outlets for investment funds at that time. By the 16th century, financial markets had developed to the point where an array of investment instruments were available: short term commercial loans, in the form of bills of exchange; bank deposits and triple contracts, also usually short term investments; long term annuities and mortgages; life annuities; and, long term and perpetual annuities issued by states and municipalities. The variety of available instruments substantially increased the need for methods of comparing the relative value of the different cash flows presented. In the case of long term investments, these calculations required recognition of compound interest or 'profit on profit'. As such, this represented a considerable evolution in religious tolerance of commercial practice as compound interest was generally deemed to be prohibited under canon law.

In addition to the growth of financial markets, the 16th century also exhibited a significant change in the social importance of business activities. This was accompanied by a shifting of the focus of economic activity to northern Europe. First Antwerp and then Amsterdam and London developed well organized exchanges dealing in a range of commodities and financial securities. The activities of these exchanges also included trading for future delivery, "time bargains", as well as options (Wilson 1941). The increasing social acceptance of business life extended only slowly into the universities, which were primarily humanist in orientation and mathematical study was largely concerned with theoretical problems (e.g., van Berkel 1988). As a consequence, many important contributions to applied mathematics and financial economics, such as the commercial arithmetics, were made outside the university system. Starting around the latter part of the 16th century in Holland, this situation began to change. Similar to early Italian contributions in commercial arithmetic, important university mathematicians were drawn to solving practical financial valuation problems, complementing the work of the commercial algorists.

Even though the development of discounting and compounding techniques were important for determining the return from partnerships and valuing commonly traded term annuities such as mortgages and lease-purchase transactions, these techniques were not sufficient to value life annuities, tontines and other types of securities involving life contingent claims. In the absence of pension funds and life insurance, life annuities performed an essential social function. The life annuity usually was a contract between three parties, the subscriber who provided the initial capital, the shareholder who was entitled to receive the annuity payments and the nominee on whose life the payout was contingent, e.g., Weir (1989). Different variations were possible: one person could be subscriber, shareholder and nominee; a parent could be a subscriber and designate a child as the nominee with the shareholder status passing from parent to child as an inheritance; or, joint life annuities could be specified where more than one nominee was designated and payments continued until both nominees died. The life annuity was further complicated by the need to establish proof of survival of the nominee prior to each annuity payment date. While it was technically possible to resell most life annuity contracts to third parties, the difficulties associated with verifying the survival and probability of survival for the nominee made resale difficult.

Until the later 17th century, market practice was to sell life annuities without taking into consideration the age of the nominee. Though there were larger and less frequent issues of life annuities by states, typical issuers of life annuities were municipalities, with prices varying widely from town to town depending on prevailing local interest rates and pricing conventions. Annuity prices were quoted in "years purchase", which is the price of the annuity divided by the annual annuity payment. For a perpetual annuity, years purchase is the inverse of the annual yield to maturity. Nicholas Bernoulli (1709) provides historical examples of life annuities selling for 6 to 12 years purchase, without allowance being made for the age of nominee.[3] De Witt (1671) quotes 14 years purchase for city of Amsterdam life annuities with a 4% interest rate and no allowance for age of nominee; this is compared with a price of 25 years purchase for a redeemable annuity, effectively a perpetual annuity with an embedded option for the borrower to redeem at the purchase price. Houtzager (1950) quotes a 16th and early 17th century Dutch pricing convention for life annuities of 1.5 to 2 times the years purchase for a redeemable annuity.

In the medieval and Renaissance periods, difficulties associated with valuing a life annuity were advantageous from the perspective of avoiding usury laws. However, by the later 17th century financial markets required more precise methods of handling the pricing risks associated with issuing life annuities. In addition to improvements in pricing techniques, different variations on the life annuity were proposed to deal with the difficulty of valuing the life contingency risk. The most important of these proposals was the tontine, a funding scheme recommended to Cardinal Mazarin of France in 1652 by Lorenzo Tonti, an expatriate Neapolitan banker living in Paris. While a number of variations were used, the generic tontine classified the subscribers' nominees into groups, by age class, creating a fund for each group. Each of the surviving persons in a group would share the interest from the fund associated with that group. When the last member of a group was dead, payments would cease. After two aborted 1653 attempts at issuing state tontines in France and Denmark, the first tontine was issued in 1670 by the Dutch town of Kampen. Following an initial issue in 1689, the tontine became an important source of state finance in France during the 18th century (Weir 1989, Alter and Riley 1986). Starting in 1693, the tontine was also used, though less extensively, for state finance in England.

By the latter part of 17th century, there were a variety of complicated fixed income securities which required valuation such as redeemable perpetual annuities, annuities with sinking funds, life annuities, lottery bonds and tontines. As financial markets evolved, another distinctly different security valuation problem emerged: pricing the issues of the earliest form of publicly traded corporation, the joint-stock company. The joint-stock form of ownership evolved somewhat slowly. Most of the early joint-stock companies retained some essential features of partnerships. Transferability was restricted in various ways, such as limiting the number of shares and requiring approval and registration of new shareholders. Many of the initial joint-stock companies were involved in long-distance trade, with paid-in capital being dispersed together with any profits after the completion of a voyage. Increases in capital were achieved by making calls on existing shareholders, rather than issuing new shares. It was not until the 1620's that joint-stock companies with the modern features started to emerge (Parker 1974). These more modern joint-stock companies included ready transferability of shares, a permanent capital stock, profits-only distributed as dividends and new capital requirements being raised by new stock issues.

Increasing supplies of both joint-stock issues and marketable government debt provided the basis for the emergence of stock exchanges. Around 1695 there were approximately 100 joint-stock companies in England with organized share trading in London, centred around Change Alley and the Royal Exchange. Organized share trading, for both English and Dutch joint-stocks and government debt, also took place on the Amsterdam Bourse. Unlike the pricing theories for fixed income securities, much of the analysis of joint-stock companies in the period under study was concerned with describing manipulative trading practices by stock-jobbers and proposing remedies for the "infamous practice", rather than with developing methods of security valuation. Fuelled by a gambling craze which "so riveted the imaginations of Europeans after 1690 that it became a metaphor for civil society itself" (Daston 1987, p.244), the excesses associated with joint stock trading culminated in the South Sea bubble of 1720; a financial market event which captures the rudimentary level of **public** understanding about security pricing which prevailed at the time.

The legacy of the South Sea bubble is comprised of two not independent parts. One part of the bubble legacy is concerned with the market manipulations arising from the use of South Sea company stock for conversion of government debt (Neal 1990). In a four month period between April and August of 1720, the manipulations led to apparently irrational price behaviour involving an increase of approximately ten times in value of South Seas Company stock, followed by an almost equally precipitous price fall in the following month. Those taken in by the fraud involved a wide range of British society, including the King and the Prince of Wales. The second part of the bubble legacy is concerned with the associated run up and collapse in prices for almost all other joint-stock issues, especially for the spate of new joint-stock issues which took place around that time. Many of these new issues were "hopelessly ill-conceived, and some downright fraudulent" (Morgan and Thomas 1962, p.37). It is estimated that in the period between September 1719 and August of 1720, 190 new issues were brought to market. Share purchases, both of South Sea stock and in other joint-stocks, were facilitated by widespread use of speculative buying with little or no margin. Positions in stock were often taken with the purely speculative objective of closing out the position prior to settlement date on the loan.

Much as with the beginning date, there are a number of possible endpoints for the early history of financial economics which could be chosen. The selection of 1776 permits the inclusion of the contributions of Abraham de Moivre (1667-1754). In addition to his seminal contributions to mathematical statistics in the 18th century, de Moivre provided fundamental results in the theoretical and practical valuation of life contingent claims and annuities. Similarly, while the contributions of Adam Smith to security pricing were not nearly as significant as in other areas of economic thought, Smith did have some strikingly modern, if largely unrecognized, ideas on the impact of agency costs on joint-stock valuation. Unfortunately, while the 1776 endpoint could also permit coverage of the contributions of Richard Price (1723-91) on the use of sinking funds for the National Debt and the introduction of old age pensions, his contributions have received considerable attention in other sources, e.g., Ogborn 1962, and brevity dictates that he not be discussed. Included in Price's other contributions are the calculation of widely used mortality and compound interest tables. In addition, Price played an important early part in the Equitable Life Assurance Society (founded 1762), the first life insurance company to utilize the principles of actuarial science in setting premiums (Ogborn 1962, Daston 1987).

2. The Early Commercial Arithmetics

A. Simple Interest In Partnerships

The Treviso Arithmetic (1478) is an untitled book published in Treviso, a town in the Venetian republic, some 26 kilometres northwest of Venice, located on the main trade route linking Venice with northern and central European centres such as Vienna and the German cities. The significance of the Treviso in the early history of financial economics lies more with what the book represents than with what it contains. The Treviso was the first printed mathematics book, though there were other potential candidates with considerably greater scholastic interest such as the works of Euclid. The book was written by some anonymous reckoning master intended as a self-study practica for those involved in commercial trades.

It is written in the Venetian dialect, not in Latin which was the language of scholarly instruction in the medieval and Renaissance universities.[4] As such, the Treviso represents the importance and advanced development that Venetian commercial education and the subject of commercial arithmetic had achieved by the latter part of the fifteenth century.[5]

Methods of calculating interest are examined in the Treviso in three problems involving the returns from partnership. No attention is given to any other situations involving interest payments. The second of these problems is as follows (Swetz, p.143):

Two merchants, Sebastino and Jacomo, have invested their money for gain in a partnership. Sebastino put in 350 ducats on the first day of January, 1472, and Jacomo 500 ducats, 14 grossi on the first day of July, 1472; and on the first day of January, 1474 they found that they had gained 622 ducats. Required is the share of each (man so that no one shall be cheated).

Observing that 1 ducat = 24 grossi and 1 grossi = 32 pizoli, the solution to this problem proceeds by applying the rule of three which, in this case, involves expressing the two contributions in grossi, 8400 grossi for Sebastino and 12014 grossi for Jacomo with the addendum that "since Sebastino has had his share in 6 months longer than Jacomo, we must multiply each share by the length of its time". Multiplying by 24 months gives Sebastino's share as 201,600 and by 18 months gives Jacomo's share as 216,252. Taking the sum of these two shares (417,852) for a divisor and applying the "rule of three" gives the solution of 300 ducats, 2 grossi, 8 pizoli and a remainder for Sebastino and 321 ducats, 21 grossi, 13 pizoli and a remainder for Jacomo.[6]

The Treviso solution to the partnership problem does not involve the use of compound interest. Using semi-annual compounding, the inclusion of compound interest would involve solving:

$$850\frac{14}{24} + 622 = 350\,(1 + \frac{y}{2})^4 + 500\frac{14}{24}\,(1 + \frac{y}{2})^3$$

The solution of y = 34.694% requires the evaluation of a quartic equation. The associated shares would be 308.4 ducats for Sebastino and 313.6 ducats for Jacomo, a decidedly different result than the 'just' result proposed in the Treviso. This failure to include compound interest was common in the early commercial arithmetics and was not due to a general ignorance of the concept. For example, Italian manuscripts dating from the 14th and early 15th century

Tuscany contained variations of the following problem: "A man loaned 100 lire to another and after 3 years he gives him 150 lire for the principal and interest at annual compound interest. I ask you at what rate was the lira loaned per month?" (Franci and Rigatelli 1988). Sometimes four year compound interest problems were proposed. The solutions proposed to these problems represent important contributions to the early development of algebra in Europe. Pegolotti (1936) provides a 14th century Italian manuscript containing tables for the compound interest calculation $(1 + i)^n$.

Two **possible** reasons can be advanced for the failure to incorporate compound interest in valuing partnership returns. Both the explanations reflect the business practices of the time. The first is simplicity of calculation. Even though the compound interest solution had been identified, the tables required for such calculations were not widely available. While a reckoning master could be consulted on the "just" solution to complicated problems, those involved in the day-to-day implementation of commercial arithmetic were primarily clerks and merchants. The Treviso algorithm, while inexact, only required applying the rule of three, a result which was at the heart of early commercial arithmetic. The second factor supporting the Treviso solution was the usury restrictions imposed by canon law. While partnerships could be used to disguise the payment of simple interest, the explicit recognition of a 'profit on profit' payment could bring the sanctions of canon law upon those requiring the receipt of such a payment. For flagrant violation, these sanctions could include ex-communication and even banishment. If such payments were made, and there is some evidence that payment of compound interest was a regular business practice at the time of the Treviso, such payments were made in silence.

B. Compound Interest

The precise impact of Chuquet's Triparty on the development of commercial arithmetic is difficult to assess. The manuscript was not published until Aristide Marre resurrected it in 1880 (Flegg, et al. 1985). As a reckoning master, Chuquet certainly passed on his insights to his students. Though there are few references to his work by 16th century mathematicians, various degrees of plagiarism were common in those days and substantial portions of his manuscript were incorporated wholesale into the 1520 publication, Larismethique nouellement composee, of Estienne de la Roche, likely a former student of Chuquet.[7] The Larismethique did have some impact, at least in France and in the important 1558/66 commercial arithmetics of another Lyon algorist, Jean Trenchant (Davis 1960). In the history of financial economics, the Triparty is important for its treatment of compound interest and for the use of the "rule of first terms" in dealing with interest calculations (Benoit 1988). Writing outside Italy, Chuquet's contributions had little apparent influence on the general study of commercial arithmetic which changed little in the 16th century, reflecting the "slowness with which the customs of commerce changed" in that period (Smith 1926).

On the subject of compound interest, the Triparty make explicit reference to the incongruity between the theoretically correct mathematical calculation and recommended commercial practice reflected in the commercial arithmetics. The manuscripts contained in the Triparty are actually three main sections concerned with algebraic theory, and three other parts containing problems, a geometry and a commercial arithmetic. The latter is generally similar in content to the Treviso, reflecting the similarity in the study of commercial arithmetic throughout Europe. However, unlike the Treviso, the handling of compound interest is examined directly:

...Three merchants formed a company, one of whom put in 10 ecus which remained there for the space of three years. The second put in 6 ecus which remained there for 7 years, and the third put in 8 ecus which remained there for four years. At the end of a period, 20 livres of profit was found. One asks how much comes to each, considering the money and the time that each has used it. (p.306)

The answer proceeds with the usual application of the rule of three as in the Treviso. After presenting this method and the solution Chuquet states:

... And the calculation is done, according to the style and opinion of some. And in order for such reckoning to be of value, it is necessary to presuppose that the principal or the capital alone has made a profit, and not the profit (itself). And inasmuch as it is not thus, for the profit and the profit on the profit made in merchandise can earn profit and profit on profit in proportion to the principal, from day to day, from month to month and from year to year, whereby a larger profit may ensue. Thus such calculations are null, and I believe that among merchants no such companies are formed. (p.307)

Hence, Chuquet holds that calculation of compound interest is the regular practice in calculating the returns from partnerships of unequal duration.

Chuquet's observation calls into question the validity of attributing the absence of compound interest problems in the early commercial arithmetics to the complexity of the solutions. The problem of determining a precise interest rate may have been more of a mathematical problem than a practical one. The main analytical difficulty in solving compound interest problems is associated with solving for the rate of interest, given the starting and ending values of the investment. However, conventional practice was to state a rate of interest, and from this the ending or starting value for an investment could be readily calculated. Customary fixed interest rates were quoted regularly, one instance being the triple contract which was often referred to as a five-percent contract (Homer and Sylla 1991, p.75). Available interest bearing securities, such as annuities, mortgages, the census and the Venetian prestiti, typically offered annual coupon payments reducing the need to deal with compounding. Where solutions were required, there were individuals with the ability to make such calculations. For example, Chuquet poses the following problem:

A merchant has lent to another a sum of money at the interest of 10%, and the interest earned like the principal at the end of every year. It happened that at the end of three years, the debtor is found to owe, as much in interest as in principal, the sum of 100 livres… determine how much had been lent to him in the first year….

Chuquet's algebraic solution to this problem provides an answer which correctly incorporates the use of compound interest.

This leaves the prohibition on usury as the primary reason that the conventional method of calculating interest derived from partnerships proposed in the early commercial arithmetics failed to account for compounding. A fundamental part of the Scholastic position on usury was derived from the Aristotle: that money is sterile and, as a consequence, it was unjust to charge for the use of money. Even though the earning of profits from investment in a partnership was acceptable under canon law, profit on the profit was not permitted (Davis 1960, p.24). Hence, even though profit on profit may have been calculated in actual transactions, it was not socially acceptable to recognize the practice in the teaching of merchant apprentices. As time progressed, this situation slowly changed. Davis (1960), for example, examines fourteen important 16th century French commercial arithmetics, all published in French, and finds 10 explain the method of computing compound interest with four of those criticizing the practice but still providing an explanation in order to be in accord with regular business practice. Similarly, the canon law prohibition on gambling made the incorporation of risk considerations into valuation problems more of a legal than an economic subject (Daston 1987, p.238-9).

3. The Development of Discounting and Present Value

As with much of early financial economics, it is difficult to trace the origins of specific valuation methods. One reason for this was a lack of attention given to these developments by the scholars of the time. Though this attitude gradually changed as commercial activities gained social importance, these activities were in the realm of merchants and, with certain exceptions such as Petrus Ramus in Germany and Rudolf Snellus in Holland (van Berkel 1988), did not warrant the close attention of true scholars. In addition, certain valuation techniques were considered proprietary by the algorist or merchant firm involved and considerable effort was made to protect trade secrets. Where scholarly contributions were involved, the widespread practice of plagiarism had a decided tendency to restrict the distribution of important developments.[8] During the 16th century, important published contributions to commercial arithmetic appeared outside of Italy, usually written in the native tongue. In Germany, Jacob Köbel (1514) and Adam Riese (1522) published the first editions of their influential works while England followed with Robert Recorde (1542) and France with Jean Trenchant (1558). Each of these publications went into numerous editions.

As financial markets and instruments developed, so did the types of problems examined in the commercial arithmetics. Tartaglia's <u>General trattato</u> (1556) contained the following problem involving the interest rate applicable to a fixed term annuity (Smith 1958, p.234): A merchant gave a university 2814 ducats on the understanding that he was to be paid back 618 ducats a year for nine years, at the end of which the 2814 ducats should be considered paid. What compound interest did the merchant earn on his money?

In commercial practice, complicated problems requiring the rate of interest as a solution were not usually encountered, making the problem primarily of interest to mathematicians. However, variation in interest rates and the widening number of fixed income securities requiring valuation created a demand for interest rate tables. Calculation of annuity payments, future and present values, given some rate of interest, were the norm. This is illustrated in L' Arithmetique (1558, 1566) by Jean Trenchant where tables for annual compounding are given for both future value, $(1+ i)^T$, and the future value of an annuity, $((1+ i)^T - 1)/i$, evaluated at i = 4% for $T \in \{1,2,3,4,5,6\}$. Another table for i= 10% and a greater than annual compounding frequency is also given (Lewin 1970).

One of the early examples involving an important university mathematician drawn to solving practical financial valuation problems, complementing the work of the commercial algorists, was Simon Stevin (1548-1620), a Flemish mathematician working in Holland, credited with introducing the decimal fraction into European mathematics (Smith 1958). In one of the chapters in La Practique d' Arithmetique (1585), Stevin goes significantly beyond Jean Trenchant in providing Tables for both present value $(1+ i)^{-T}$ and present value of annuities $((1+ i)^T - 1)/(i(1+ i)^T)$ for $T \in \{1,2,3.....30\}$ and $i \in \{1\%,2\%....16\%\}$. Stevin also demonstrates the relationship between present value and future value, for both single cash flows and annuities. Finally, Stevin demonstrates the method of using the Tables for doing yield to maturity calculations in term annuity problems such as: "Someone owes £1500 p.a. to be paid over the next 22 years, and he pays his creditor £15,300 in lieu; what rate of interest does this represent?" However, Stevin does not consider less than annual compounding, providing a number of arguments against the practice. Further developments in the area of solving interest valuation problems were also made by commercial algorists, such as the Englishman Richard Witt who published Arithmetical Questions, touching the Buying and Exchange of Annuities… (1613) (Lewin 1970). This book provides present value, present value of annuity and future value of annuity Tables for $T \in \{1,2,....30\}$ with a 10% rate of interest, which was the prevalent rate in England at that time. Future value tables are also provided for a range of interest rates. Witt goes beyond Stevin in considering less than annual compounding frequencies.

A number of significant implications can be drawn from the widespread availability of present and future value interest tables. The detailed number of calculations contained in the available tables reflected substantive changes in security valuation practices brought on by the economic and social transformation that was coincident with the Reformation. Rationalization in the financial exchange process, brought on by a deepening and widening of capital markets, meant that more accurate pricing was required in order to adequately compare investment opportunities. Trading activity in security markets was facilitated by the financing demands of local and national governments and the emergence of the monied commercial and rentier classes seeking investment outlets. Even though church doctrine was slow to change, prohibitions on usury were by the 17th century primarily aimed at extortionate practices disadvantaging the weak members of society. Easing of prohibitions against usury progressed more rapidly in Protestant than Catholic countries, leading to more rapid evolution of security pricing practices in Holland and, especially, England. This development provided the requisite preconditions for substantive developments in security pricing techniques.

4. The Valuation of Life Annuities

One of Simon Stevin's lesser known contributions involved drawing up the statutes and curriculum for a new mathematical school for engineers which was created at Leiden in 1600 (van Berkel 1988). The focus of the mathematical instruction was decidedly more practical than conventional university instruction and it was in Leiden that Frans van Schooten (1615-1660) instructed a number of important students including Christian Huygens (1629-1695), Jan Hudde (1628-1704) and Jan (Johan) de Witt (1625-1672). Aided by contributions from Huygens and Hudde, in Value of Life Annuities in Proportion to Redeemable Annuities (1671, in Dutch) de Witt provided the first substantive analytical solution to the difficult problem of valuing a life annuity.[9] Unlike the fixed term annuity problems which had been solved in various commercial arithmetics, the life annuity valuation required the weighting of the relevant future cash flows by the probability of survival for the designated nominee. De Witt's approach, which is somewhat computationally cumbersome but analytically insightful, was to compute the value of a life annuity by applying the concept of mathematical expectation advanced by Huygens in 1657.[10]

De Witt's approach involved making theoretical assumptions about the distribution of the number of deaths. To provide empirical support for his calculations, he gave supplementary empirical evidence derived from the register at The Hague for life annuitants of Holland and West Friesland for which he calculated the average present values of life annuities for different age classes, based on the actual payments made on the annuities. This crude empirical analysis was buttressed by the considerably more detailed empirical work of Hudde on the mortality statistics of life annuitants from the Amsterdam register for 1586-90. Like de Witt, Hudde also calculates average present values for all annuitants and for the 1 to 10 year age class, coming up with values similar to de Witt. Though Hudde's work has been recognized by statisticians as an important early contribution to the calculation of life tables, the value of the statistical contribution is limited because the data he presented were based only on the lives of life annuity nominees and, as a result, are not representative of the whole population. However, as contributors to the history of financial economics, the theoretical work of de Witt and the empirical work of Hudde must be considered **seminal**. Important elements of modern financial economics, such as contingent claim pricing and risk neutral valuation, are reflected in this early work.

Much like Chuquet, it is difficult to assess the impact of de Witt's contribution to the pricing of life annuities. Based on his recommendation, in 1672 the city of Amsterdam began offering life annuities with prices dependent on the age of the nominee. However, this practice did not become widespread and by 1694, when Edmond Halley (1656-1742) published his influential paper "An Estimate of the Degrees of Mortality of Mankind....", the English government was still selling life annuities at seven years purchase, independent of age. Halley's "Estimate..." is primarily concerned with presenting a life table calculated from the detailed birth and death registers of Breslau in Silesia. At the time, the most important source of statistical demography was John Graunt's Natural and Political Observations Made Upon the Bills of Mortality (1662) which was limited by the incomplete records which the bills of mortality for London provided, e.g., Pearson (1978, Chap.II). Halley's "Estimate..." represents a seminal contribution to statistics and actuarial science and touches annuities only as an illustration of applying the information in the life table. In the process, Halley presents a somewhat different approach than de Witt to the valuation of a life annuity. In fairness to de Witt, Halley's contribution also did not deter the English government from continuing to offer life annuities at seven years purchase, without reference to age.

In assessing Halley's contribution to the history of financial economics, it is difficult not to mention Abraham de Moivre (1667-1754), an expatriate Frenchman transplanted to London following the Repeal of the Edict of Nantes. Halley and de Moivre were first acquainted in 1692 and in 1695 de Moivre's first paper contributed to the Royal Society was presented by Halley. Unlike Halley who touched once, and only briefly, on the pricing of securities, de Moivre spent much of his productive life concerned with pricing life annuities. Never able to secure an academic position, de Moivre earned a living as an 18th century reckoning master and algorist, tutoring mathematics, calculating odds for gamblers and reckoning values for underwriters and annuity brokers (Pearson 1978, Chp. VI). It is not difficult to conceive enlightened interaction between the two on the subject of applying Halley's life table and de Moivre suggesting and explaining the important problem of life annuities. De Moivre's primary contribution to pricing life annuities was Annuities Upon Lives (1725) with a second edition (1743). Also important is the 1756 edition of his Doctrine on Chances which contains a section titled "A Treatise of Annuities on Lives" together with discussion of the life tables of Halley, Kersseboom, Simpson and Deparcieux.

In Annuities, de Moivre examined a wide variety of the life annuities available in the early 18th century: single life annuities, joint annuities (annuities written on several lives), reversionary annuities, and annuities on successive lives. His general approach to these valuation problems involves two steps: first, to develop a general valuation formula for each type of annuity based on Halley's approach; and, secondly, to produce an approximation to the general formula suitable for calculating prices without the considerable efforts involved in evaluating the more exact formula. In order to implement some of the approximations, de Moivre developed a mathematical formulation, a piecewise linear approximation, of the information contained in the life table. The computational advantages of de Moivre's approximations were considerable and the methods became widely used in day-to-day commercial practice. However, the ensuing development of actuarial science and insurance mathematics progressed by working with the more tedious exact formulae, estimating more accurate life tables and calculating tables with exact prices for different situations and levels of interest rates.[11]

Because most of the substantive problems in the exact theory of life annuities had been solved by de Moivre, subsequent initial contributions to the valuation of life annuities were primarily empirical and computational. Thomas Simpson (1710-1761) produced The Doctrine of Annuities and Reversions (1742) and The Valuation of Annuities for Single and Joint Lives (1752) which calculate a number of useful valuation tables for both single and joint lives using different rates of interest.[12] Other substantive contributions were made by the Dutchmen Nicholas Struyck (1687-1769) and Willem Kersseboom (1691-1771), the former a mathematician and the latter a statistician. In 1738, Kersseboom published an article in the spirit of Hudde's work on life tables for annuitants and provides a valuation table for single life annuities. Struyck also examines the valuation of life annuities in a memoir which is part of Introduction to General Geography, besides certain astronomical and other memoirs (1740, in Dutch). Similar to Hudde and Kersseboom, Struyck recognizes the importance of basing valuation tables on life tables for annuitants and not on life tables for the general population as Halley did. Using the registers for the Amsterdam life annuitants for 1672-74 and 1686-89, Struyck is the first to construct separate valuation tables for males and females.

In his Essay on the Probabilities of the Duration of Human Life (1746, in French) Antoine Déparcieux (1705-1768) acknowledges and extends the work of Kersseboom to lists of annuitants for the French tontines of 1689 and 1696. He conclusively demonstrates the atypical mortality of the French rentier, compared to both general populations as in Halley's life table and to Kersseboom's Dutch annuitants (Pearson 1978, p.200). Based on his life table, he provides numerous tables calculating the present values of various types of life annuities: single life annuities, joint annuities, tontines, compound tontines and so on. Deparcieux was careful to provide accurate explanations of his life tables and present value calculations, a feature which distinguishes his work from similar efforts around the same time.

Deparcieux continued the work of the Essay with Addition à l' Essai sur les probabilitiés de la durée de la vie humaine (1760) which provides the first available tables for the value of postponed life annuities, fundamental to evaluating pension fund cash flows. From this point it can fairly be argued that contributions to the study of life annuity valuation were concerned with application, extension, improvement and clarification, rather than in producing initial theoretical pricing results.

While it is appealing to conclude that the substantial theoretical development in the pricing of aleatory contracts was accompanied by a similar improvement in commercial practices, this was not the case. In the area of insurance, this was partly due to legal and religious prohibitions on gambling. England, perhaps the most progressive nation in incorporating theoretical advances into commercial practice, did not establish the legal preconditions for an insurance industry until the Gambling Act of 1774. Up to this point, many insurance schemes were targetted more at gambling outcomes than risk reduction. The small number of reputable insurance companies did not use actuarial principles in determining either premiums or payouts. Premiums were usually charged at a flat rate per period and payouts determined by dividing the available premium pool between eligible claimants for that period. The first insurance company to apply actuarial techniques was the Society for the Equitable Assurance on Lives and Survivorships (estab. 1762). Much as in modern insurance, the Equitable established premiums based on age, created a fund with which to make future claims and provided for a guaranteed fixed payout in the event of claim.

The resistance of market practice to adopting theoretical pricing results is also reflected in the pricing of life contingent claims. In pricing tontines and life annuities prior to the French revolution, the French government demonstrated only limited ability to set actuarially accurate prices, though Weir (1989, p.118-19) attributes this to factors other than ignorance, such as the desire to disguise the true cost of the debt. Another rationale for underpricing life annuities was political: to provide a retirement subsidy for the increasingly powerful urban bourgeoisie, the primary purchasers of the government' s life contingent debt. These motivations may also partly explain the French and English government practice of not accurately accounting for the age of the nominee in pricing life annuities. For example, even though the English government demonstrated much better understanding of theoretical pricing for life annuities and tontines than the French government, when the English government ended the obviously inaccurate practice of issuing life annuities without regard to age in the latter part of the 18th century, mispricing associated with the practice of permitting selection of nominees continued until 1852.

5. Pricing Theories for Joint Stocks and Derivative Securities

A. Joint Stock Companies

Unlike the pricing of life contingent claims, by the 18th century the analysis of joint-stocks and derivative securities exhibited little progress beyond descriptive pamphlets such as John Houghton's 1694 contribution to his weekly journal A Collection for the Improvement of Husbandry and Trade (1692-1703) or Daniel Defoe The Villany of Stock-Jobbers detected (1701). In the case of derivatives the lack of analysis is understandable because, even though trading in commodity options and time bargains pre-dates the emergence of joint-stocks, e.g., Wilson (1941), the activity was carried on by a specialized class of trader and was not widely understood or observed. The same cannot be said for joint-stock companies, which played an important role in both commercial activities and state finance.[13] Joint-stock companies possess two essential features not embodied in the typical partnership: transferability and limited liability. While the transferability feature of joint-stock issues can be traced to transferable partnership shares in 15th century Italy, the combination of partial limited liability and transferability is usually first associated with the establishment of the Russia Company in England in 1553 (Morgan and Thomas, p.12).

Unlike fixed income securities, the cash flows associated with stocks are much less predictable making valuation a more uncertain exercise. Even modern financial economics lacks a **theoretical** model of stock pricing with any practical accuracy, offering in its place a theory of portfolio management based on quadratic optimization in which the determination of individual stock prices is not directly addressed. The modern approach to the valuation of individual stocks relies on a qualitative analysis of fundamental information. Even though it is written as a descriptive dialogue of the activities of the Amsterdam Bourse in the later 17th century, Joseph de la Vega Confusion de confusiones (1688) demonstrates a modern understanding of the use of fundamental information to value stocks:

The price of shares (in the Dutch East India Company) is now 580... it seems to me that they will climb to a much higher price because of extensive cargoes that are expected from India, because of the good business of the Company, of the reputation of its goods, of the prospective dividends and of the peace in Europe. (Confusion, p.156)

Recognizing the uncertainties in seaborne trade and the difficulty in obtaining information about incoming cargoes, de la Vega goes on to describe how some traders could profitably trade on information about incoming cargoes from the East. He correctly recognizes that such information alone is insufficient but would depend also on European conditions and the safe arrival and unloading of cargo.

Even though de la Vega identifies how the price of joint stocks can be determined by fundamental information, much of his dialogue is taken up in a description of how prices will deviate from the fundamental values based on the expectations of bulls and bears. The last of the four dialogues is concerned with detailing methods of market manipulation: "...the acme of Exchange operations, the craftiest and most complicated machinations which exist in the maze of the Exchange and which require the greatest possible cunning" (Confusion, p.191). The manipulation of securities markets in the 18th century was facilitated by the social practice of using securities for purposes of gambling. This practice was in keeping with the widespread public acceptance of gambling reflected, for example, in the use of lotteries to increase the attractiveness of government debt operations (Cohen 1953). However, gamblers were not the only participants in the stock markets:

...it should be observed that three classes of men are to be distinguished on the stock exchange. The princes of business belong to the first class, the merchants to the second, and... gamblers and speculators to the third class.

Recognizing that the motives of gamblers and speculators were often somewhat nefarious, there is considerable evidence that valuation methods for joint-stocks making using of fundamental information were in general use by brokers to support their activities relative to de la Vega's first and second types of participants. For example, Wilson (1941, p.124) quotes a **1720** correspondence between the London attorney and stock broker, Peter Crellius, and David Leeuw, a Dutch investor: "Shares seem to be notably higher, but it looks to me as if the best-informed people are against the rise and great projects of the South Seas Company, believing the Bank and East India Company to be, in general, more secure and reliable."

The widespread social use of joint-stocks for gambling purposes was an important precondition to the South Sea bubble. The two opposing interpretations of the bubble, and market manias in general, reflect fundamentally different views regarding security pricing behaviour: one views maintains that security markets are driven by capricious investors generating the "extraordinary popular delusions" associated with numerous bubbles and manias. The other view maintains that prices in security markets are inherently rational and bubbles and other apparently irrational market phenomena are due to market manipulation or institutional failure (Garber 1990, p.16-7). On the subject of manias versus manipulation, Richard Cantillon (1685?-1734) provided an important perspective. Cantillon was a successful, if somewhat shady, banker whose contribution to the development of political economy was overlooked for many years, being rediscovered years later by W. Stanley Jevons (1881). Though he never directly addresses the causes of the South Sea Bubble, the Essai sur la Nature du Commerce (1725?, published 1755) was written during the period of the worst excesses. (Jevons refers to the Essai as "the Cradle of Political Economy".) The Essai is decidedly in favour of manipulation as a necessary factor in observed irrational pricing behaviour.

Like the early reckoning masters and other important contributors to early financial economics such as Chuquet, de Witt and de Moivre, Cantillon drew on his commercial experiences to motivate his analysis.[14] In Chap. VIII, Part III of the Essai Cantillon provides a significant insight into early Bank of England operations in the government debt market:

> If the Bank alone raises the price of public debt stock by buying it, it will by so much depress it when it resells to cancel its excess issue of notes. But it always happens that many people wishing to follow the Agents of the Bank in their operations help to keep up the price. Some of them get caught for want of understanding these operations, in which there enter infinite refinements or rather trickery...(p.323)

Having recognized the important role of participation in the markets by uninformed traders, Cantillon goes on to observe:

It is then undoubted that a Bank with the complicity of a Minister is able to raise and support of the price of public stock and to lower the rate of interest in the State… and thus pay off the State debt. But these refinements which open the door to making large fortunes are rarely carried out for the sole advantage of the State, and those who take part in them are generally corrupted. (p.323)

This statement appears as a somewhat veiled as a generality, in close proximity to a discussion about the **Bank of England's** manipulative debt market actions. Cantillon continues with the following statement: "…if some panic or unforeseen crisis drove the holders (of banknotes) to demand silver from the Bank the bomb would burst and it would be seen that these are dangerous operations." In this connection, it is possible that Cantillon was making referring to the collapse of John Law's system in France or to the role the Bank of England may have played in the South Sea bubble. There is no doubt that he is accusing the directors of the Bank, with the acquiesence of the government, of engaging in debt market manipulation for their own self-enrichment.

An important 18th century analysis comparing of joint-stock companies is provided by Adam Smith in The Wealth of Nations (Bk.V, Ch.1, Pt.III, Art. 1).[15] Smith explicitly recognizes the difficulties inherent in the valuation of shares in joint-stock companies: "The value of a share in a joint stock is always the price which it will bring in the market; and this may be either greater or less, in any proportion, than the sum which its owner stands credited for in the stock of the company" (p.232, Canaan 6th Edition). Smith goes on to provide a significant analysis of joint-stock companies as sources of corporate finance. Smith begins by contrasting the joint-stock company with a partnership, recognizing the features of transferability and limited liability. Transferability brings with it the risk that, at sale, the value received will not equal "his share of the common stock" or retained earnings plus paid-in capital. This is in contrast to partnerships where shares are not usually transferable and "upon proper warning" a partner may withdraw and receive his appropriate share. In addition to the market price risk associated with transferability, Smith also identifies the ability to transfer joint-stock shares to another person "without (the) consent" of the other members of the company.

Having recognized the essential features of transferability and limited liability, Smith proceeds to construct an indictment of the usefulness of the joint-stock form of organization for all but a restricted list of economic activities. The crux of his argument depends on the modern notion of **agency costs**:

...The directors of (joint-stock) companies...being the managers rather of other people's money than of their own, it cannot well be expected, that they should watch over it with the same anxious vigilance which the partners in a private copartnery frequently watch over their own.... Negligence and profusion, therefore, must always prevail, more or less, in the management of the affairs of such a company. (p.233)

In making this argument, Smith also recognizes that the "general court of proprietors" or board of directors "seldom pretend to understand any thing of the business of the company; and when the spirit of faction happens not to prevail upon them, give themselves no trouble about it, but receive contentedly such half yearly or yearly dividend, as the directors think proper to make to them" (p.232). Recognizing that joint-stock issues are capable of raising significantly larger amounts of capital than partnerships, Smith concludes that there are only four types of business for which joint-stock is an acceptable form of organization: banking, insurance, canal building and water works.

Writing before the advent of the Industrial Revolution, Smith's views on joint-stock companies were conditioned by the performance of those companies up to his time. This included the dealings of the South Sea Company which contributed to the South Sea bubble:

The South Sea Company never had any forts or garrisons to maintain... But they had an immense capital divided among an immense number of proprietors. It was naturally to be expected, therefore, that folly, negligence, and profusion should prevail in the whole management of their affairs. The knavery and extravagance of their stock-jobbing projects are sufficiently known, and the explication of them would be foreign to the present subject. Their mercantile projects were not much better conducted. (p.235-6)

It is unfortunate that Smith did not attempt a detailed discussion of his views on the "stock-jobbing projects" of the South Sea Company. Despite numerous, seemingly exhaustive studies, the causes of the South Sea bubble are still a subject of debate, e.g., Neal (1990).

B. Derivative Securities

One of the most interesting, unanswered question in the early history of financial economics concerns the methods used for pricing derivative securities transactions, particularly options and time bargains. Trading for deferred delivery, similar to forward contracting, has a history going back to antiquity, associated with transactions involving long distances, slow transport and poor communications. The uncertainties associated with the quality and timing of delivery led naturally to embedding options into contracts. As commodity markets developed, the deferred delivery and options features of the transactions were gradually securitized. Trading in both commodity time bargains, an early form of forward contract, and options was well-developed on the Antwerp Bourse by the time the Treviso was published. The bulk of these derivatives transactions were concerned with goods involved in seaborne trade, making it difficult to identify whether the transactions were initially purely speculative or were motivated by hedging considerations. However, over time, participation by traders with purely speculative motives became considerable.

Following the collapse of Antwerp in 1585, much of the commodities trading business shifted to Amsterdam where trading in derivatives was refined substantially. The information that is available about the trading of derivative instruments on the Amsterdam Bourse, e.g., de la Vega's Confusion (1688) and de Pinto's Jeu d'Actions en Hollande (1771), indicates these securities were used primarily for speculating and not for purposes of risk management. Almost from the beginning of trade on the Bourse, the speculative aspects of trading attracted the attention of the Dutch authorities. Following a speculative "bear raid" in 1609 involving uncovered long-dated short sales of Dutch East India company shares by a group of speculators led by Isaac le Maire, speculative trading involving uncovered positions was banned in 1610. While violation of the ban did not lead to prosecution, it effectively removed the protection of the courts for the purposes of enforcing the contracts. This left enforcement of contracts up to the individual brokers involved. While it was possible to repudiate a losing position, available sanctions involved exclusion from trading on the Bourse, a sanction sufficiently severe to ensure that brokers would settle all but the most substantial losing positions.

Despite the ban on trading in uncovered positions, the development of cash market trading in joint-stocks was associated with similar progress in derivatives trading. By 1630, speculative trading had progressed to the point where gains or losses on positions were settled on rescontre (settling day) without delivery of the cash securities, and positions could be carried forward to the next rescontre. By the late 17th century a regular monthly (changing to quarterly) rescontre process was in place. Derivatives trading also spilled over into other areas of Dutch economic life, leading in one instance to the tulipmania of 1635-37.[16] Trading in options and time bargains on joint-stocks had spread to London by the end of the 17th century (Morgan and Thomas, p.59-64), inheriting the essential features of derivatives trading conducted in Amsterdam. Much as in Holland, various legislative attempts were made to restrict or prohibit derivatives trading culminating in Barnard's Act of 1734 which banned trading in options and speculative time bargains. As in Holland, this did not prevent the trading of derivatives but, rather, made brokers the principals in derivatives transactions, liable for any settlement failure on the part of clients.

In the absence of a primary source directly concerned with the methods of pricing of derivative securities, it is still possible to infer that while prices were, at times, determined by forces of supply and demand, there was also some understanding and application of the concept of cash-and-carry arbitrage:[17]

> Speculative buyers paid to sellers the percentage by which funds had fallen since the last contango day or alternatively received.... After surpluses had been paid, new continuations were undertaken for the following settlement. In such a prolongatie (continuation) the buyer granted the seller a certain percentage (a contango rate) to prolong his purchase to the next rescontre... The prolongatie was charged for at a rate based on dividends which the funds bore. But if there were many speculators a la hausse (bulls) the contango rate become proportionately dearer, bringing a clear advantage to sellers. Conversely, a big proportion of sellers reduced the contango rate. Under the pressure of political events in 1755, the "backwardation" rate appeared, paid by speculators a la baisse (bears) for the privilege of deferring delivery of the stock sold. (Wilson p.83-4)

The typical contango in prices for time bargains was associated with the requirement that the seller would make any relevant dividend payments to the buyer. "The basic contango rate for 4% Annuities was 1% for each rescontre, coming to 4% per annum: East India funds bearing 6% had a 1 1/2% contango rate." (Wilson, p.85)

In normal markets, it was not possible to make arbitrage profits by borrowing money in order to purchase funds, receiving the dividends and selling the funds forward at the next rescontre (turning the dividend payments over to the purchaser). However, "when opinion was optimistic (and) prolongers had to pay for time and hope, and 1% became 2 or 3%" (Wilson p.85), then there were potential arbitrage opportunities and, it appears, arbitrage trading did take place, providing the necessary market liquidity to clear the positions required by speculators. Unlike time bargains, arbitrage requirements seem to have had less impact on option prices. Wilson (p.122), for example, provides quotes for options on East India Company and South Sea Company shares in 1719 which reflect some pricing inefficiencies. Option prices reflect a general pricing advantage for writers, consistent with the view that most buyers were "out-and-out gamblers". Option writers quoted prices at premiums consistent with exploiting market sentiments. The tendency of options trading, at least in England, to be concentrated among less reputable brokers (Morgan and Thomas, p.61-2) and to be associated with market manipulation also argues against sophisticated understanding of option pricing.

6. Some Speculative Conclusions

The relative absence of studies on the early history of financial economics begs an obvious question: what is to be gained from an examination of the history? Despite being written primarily as an overview, this paper demonstrates that there are numerous unexplored issues from which there is a considerable amount to be gained. For example, consider the question: why has the history of financial economics been relatively ignored? The search for a resolution of this question in the early history leads to the work of Petrus Ramus and Rudolf Snellius where the subject matter of university teaching is seriously questioned as being overly complicated, theoretical and abstract (van Berkel 1988). The implication is that the absence of historical studies on financial economics is, at least partly, due to an inherent bias in university instruction and research. Financial economics is 'use' oriented and, as such, was not an interesting or accessible subject to those involved in generating historical research. For similar reasons, it is hard to trace the genealogy of certain contributions because those involved were more concerned with the use of the ideas rather than with correctly recording the process of production. In some cases, the proprietary character of the ideas dictated against accurate recording.[18]

Despite a relative dearth of primary materials, in the end it is still possible to recognize a number of seminal early contributions to financial economics. The beginnings of modern contingent claims analysis can be attributed to the Dutchmen Jan de Witt and Jan Hudde, neither man being either a financial market practitioners or a scholastics. While recognizing Chuquet's early contributions to solving compound interest problems, much of the analytical foundation for modern fixed income analysis can still be credited to Abraham de Moivre. Although he was concerned primarily with the now out-dated problem of pricing life annuities, de Moivre was the first to introduce important mathematical techniques, such as series solutions, to the pricing of complicated fixed income securities. In the area of joint-stock valuation, in addition to de la Vega's descriptive insights, Adam Smith can be credited with an insightful and surprisingly modern presentation of agency costs. Finally, while there were a number of descriptive and polemical studies, there is relatively little in the early history of security valuation concerning options and forward contracts to warrant a specific mention.

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NOTES

1. Useful tables outlining the date, type and geographical sources of the various types of fixed income securities examined in this paper can be found in Homer and Sylla (1991): p.103 for the Venetian prestiti from 1299-1403 and p. 109 for 1399-1502, for other types and geographical locations, p.102-3 for the 14th century, p.110 for the 15th century, p.120-1 for the 16th century, and p.131 for the 17th century.

2. Certain bequests arising with inheritances, such as maintenances and life incomes, can have cash flows similar to census agreements. Such bequests go back to prehistoric times. In Roman times, the value of a life annuities relative to a term annuity was given by the jurist Ulpian. His rule proposed that a life annuity be equated with a term annuity where the term was equal to the annuitant's life expectation. This valuation attracted the attention of Nicholas Bernoulli who demonstrated that the Ulpian valuation has a significant upward bias for the price of a life annuity (Homer and Sylla 1991, Hald 1990).

3. Daston (1987, n.5) quotes James (1853) for a 14% rate on English life annuities, for any age, issued by the state under William III. This translates into approximately 7 years purchase.

4. Other significant features of the Treviso include: the book was published in the Venetian republic and not in Germany, the region where printing originated around 1450 with Gutenberg and Fust; and, probably due to its lack of scope and the emergence of other more comprehensive commercial arithmetics, the book was not successful and it did not survive to a second printing. Copies of the book are, today, very rare. Swetz (1987) provides further discussion.

5. This does not mean that the Treviso was the most important commercial arithmetic of the time. While the Treviso was the first published, from 1480-1490 at least 63 mathematical works were printed in Italy, mostly commercial arithmetics. Of these, the first to have considerable importance was the Libro de Abacho by Pietro Borgo (Borghi) which went into 16 editions, spanning almost one century. The Summa de arithmetica, geometria, proportioni e proportionalità (1494) by Fra Luca Paccioli (Paciolo) is often identified as the most important academic Italian work of this period, though the Summa is largely a comprehensive summary of the mathematical knowledge of that time (David 1962, p.32). The importance of commercial arithmetic is reflected in the Summa by the considerable amount coverage given to this material. The Summa is also recognized as the most comprehensive treatment of double entry bookkeeping available during this period (de Roover 1974).

6. The rule of three is basic to problems involved in exchange. The rule was considered so important that it was often referred to as 'the golden rule'. The rule involved the different variations on proportions: a is to b as c is to x, using a, b and c to solve for x. The different variations on the rule were typically taught in rote fashion, without reference to the basic algebraic foundation.

7. The Larismethique is considered to be "vastly inferior" to the Triparty by Marre who was concerned primarily with Chuquet's contributions to algebra.

8. An important example of plagiarism occurred surrounding the solution of the cubic and quartic equations. Around 1525 Nicolo Fontana, better known as Tartaglia, devised a method for solving a wide range of cubic equations, at a time when only a restricted number of cubic equations could be solved. Under considerable pestering by the prolific Girolamo Cardano, in 1537 Tartaglia partially revealed his secret solution procedure, swearing Cardano to secrecy. When one of Cardano's students, Ludovico Ferrari, developed a solution for a quartic equation in 1540 using the procedure, the result was one

of the bitterest scholastic disputes of the 16th century. The debate was sparked by the 1545 publication of Ferrari's result in Cardano's Artis Magnae Liber where Cardano passes the quartic result off as his own (David 1962).

9. Unlike Huygens, both Hudde and de Witt were amateur mathematicians (Coolidge 1990). Jan de Witt had a distinguished political career, which included being the Grand Pensionary (prime minister) of Holland. It was during his time as prime minister that he produced his Value of Life Annuities… which was written for the purpose of explaining how government could accurately price life annuities to raise funds for an impending war with France. When France invaded in 1672, de Witt resigned as prime minister. In the ensuing public panic which swept Holland shortly after, he was shot, publicly hanged and had his body violated by a mob. Karl Pearson, who had strong views on a number of individuals involved in the history of statistics, depreciates de Witt's work by claiming: "…the data are uncertain and the method of computation is fallacious" (Pearson 1978, p.100). This is at variance with Hald (1990), Alter and Riley (1986) and others.

10. There are various sources on the valuation of life contingencies, e.g., Alter and Riley (1986), Hald (1990) and Pearson (1978).

11. In addition to work on life annuities and related problems, the traditional problem of calculating present value and future value interest tables continued. In 1726, John Smart published his comprehensive Tables of Interest, Discount and Annuities. These tables, taken to 9 significant figures, are credited (Pearson 1978) with being the primary source for tabular interest calculations in de Moivre, Simpson and others.

12. Simpson is perhaps better known for being accused, in numerous sources (e.g., Hald 1990, Pearson 1978), of shamelessly plagiarizing the contributions of de Moivre, both on life annuities and in probability theory. Simpson also took liberties with the contributions of other writers such as John Smart.

13. This statement is not meant to imply that the holders of joint-stocks were numerous. On the contrary, there were a relatively small number of individuals involved. For example, in 1691 the combined stock of the East India and Africa Companies was divided into 680 holdings (some held by the same person). For both English and Dutch joint stock issues, most of the holders of joint-stock lived in London or Amsterdam (Parker 1978, p.559).

14. Like de Moivre and Chuquet, Richard Cantillon was also the subject of plagiarism, in this case by a relative Phillipe Cantillon (Jevons 1881). The theme of plagiarism also occurred with Kersseboom and Struyck, both accusing the other.

15. Smith was not the first to deal with the problems of the joint-stock form of ownership. For example, the problems of inefficient production associated with "stock-jobbing management" were raised in Parliamentary enquiries going back to at least 1696 (Morgan and Thomas, p.22-3). Smith also references a number of earlier works on joint-stock companies such as Abbe Morellet, Examen de la Reponse de M. Necker (1769) and, especially, Adam Anderson, The Historical and Chronological Deduction of the Origin of Commerce.

16. Garber (1990) examines to what extent there really was a tulipmania. The bulk of irrational pricing appears to have been associated with tavern trading of unenforceable contracts: "The authorities did not prosecute people for participating in proscribed futures contracts. They simply refused legal enforcement of such contracts.… The futures trading, the centre of the (tulipmania) activity, was clearly banned by the edicts; and, in the end, the courts did not enforce deals made in the (taverns), all of which were repudiated. It is incomprehensible that anyone involved in the fluctuating associations of the taverns

would have entered such unenforceable agreements in the first place unless they were merely part of a game." (Garber, p.19) Schama (1987) also provides a detailed discussion of the tulipmania and its social underpinnings.

17. Wilson (1941, Chap.III (iii) and Chap.IV (v)) provides a useful summary of de la Vega, de Pinto and some correspondence between David Leeuw and Peter Crellius.

18. The neglect by historians of 'use' oriented subjects is not systemic. In recent years, historians of science have exhibited a growing interest in the history of practice. Daston (1987, 1988) are two excellent examples of this line of research.